Kutta-Joukowski theorem

The lift force per unit span of the airfoil, F_L', can be computed as

$$F_L' = \rho \vec{u}_\infty \times \vec{\Gamma}, \qquad \text{(K-J.1)}$$

where the direction of $\vec{\Gamma}$ is perpendicular to the two-dimensional plane of the flow. It is related to the sense of rotation of potential vortex / vortices by the right-hand rule.

The magnitude $\Gamma = \vec{\Gamma} \cdot \vec{e}_z$ is called "circulation". It is defined as

$$\Gamma = \oint \vec{u} \cdot \vec{t}\, ds, \qquad \text{(K-J.2)}$$

where the integral is taken over any arbitrary path (lying in the plane of the flow) outside the airfoil. That is, Γ is independent of the integration path. $\vec{t}$ is a unit tangent vector to the integration path.

Γ is also the strength of a potential line vortex, which induces the circulation.

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The circulation induced by a potential vortex sheet is obtained by integration of the vortex strength distribution

$$\Gamma = \int \gamma(s)\, ds . \qquad \text{(K-J.3)}$$

Lift coefficient

The lift coefficient is defined as

$$C_L = \frac{F_L}{\frac{1}{2}\rho|\vec{u}_\infty|^2 A} , \qquad \text{(K-J.4)}$$

where F_L is the total lift force, and A is some characteristic area, e.g., the area of the airfoil.
For 2D flow, the lift coefficient is defined in terms of the lift force per unit span, F_L', and the streamwise length (chord) of the airfoil, L, as follows:

$$C_L = \frac{F_L'}{\frac{1}{2}\rho|\vec{u}_\infty|^2 L} . \qquad \text{(K-J.5)}$$

$\boxed{\text{K-J2}}$

Substituting Eq. (K-J.1) into Eq. (K-J.5), the lift coefficient is related to the circulation as follows:

$$C_L = \frac{2\Gamma}{|\vec{u}_\infty| L}. \qquad (K\text{-}J.6)$$

The ratio $\Gamma / |\vec{u}_\infty| L$ is the dimensionless circulation corresponding to a unit length of the airfoil and unit far-fiel velocity.