

Potential

In[3]:= `Integrate[xi*Log[Sqrt[(x-xi)^2+y^2]],{xi,-1/2,1/2},Assumptions->{x ∈ Reals, y ∈ Reals, y≠0}|x`

$$\text{Out[3]} = \frac{1}{2} \left(x \left(-1 + 2y \left(\text{ArcTan} \left[\frac{\frac{1}{2} - x}{y} \right] + \text{ArcTan} \left[\frac{\frac{1}{2} + x}{y} \right] \right) \right) + \left(-\frac{1}{4} + x^2 - y^2 \right) \text{ArcTanh} \left[\frac{4x}{1 + 4x^2 + 4y^2} \right] \right)$$

U

In[8]:= `Integrate[xi*(x-xi)/((x-xi)^2+y^2),{xi,-1/2,1/2},Assumptions->{x ∈ Reals, y ∈ Reals}]`

$$\text{Out[8]} = -1 + y \left(\text{ArcTan} \left[\frac{\frac{1}{2} - x}{y} \right] + \text{ArcTan} \left[\frac{\frac{1}{2} + x}{y} \right] \right) + x \text{ArcTanh} \left[\frac{4x}{1 + 4x^2 + 4y^2} \right]$$

if $y \neq 0 \mid \mid x < -\frac{1}{2} \mid \mid 2x > 1$

V

In[13]:= `Integrate[xi*y/((x-xi)^2+y^2),{xi,-1/2,1/2},Assumptions->{x ∈ Reals, y>0}]`

$$\text{Out[13]} = x \text{ArcCot} \left[\frac{2y}{1 - 2x} \right] + x \text{ArcTan} \left[\frac{\frac{1}{2} + x}{y} \right] + \frac{1}{2} y \text{Log} \left[1 - \frac{8x}{(1 + 2x)^2 + 4y^2} \right]$$

In[10]:= `Integrate[xi*y/((x-xi)^2+y^2),{xi,-1/2,1/2},Assumptions->{x ∈ Reals, y<0}]`

$$\text{Out[10]} = x \text{ArcCot} \left[\frac{2y}{1 - 2x} \right] + x \text{ArcTan} \left[\frac{\frac{1}{2} + x}{y} \right] + \frac{1}{2} y \text{Log} \left[1 - \frac{8x}{(1 + 2x)^2 + 4y^2} \right]$$