Remarks: (S&G, p. 148-149)

- B.L. eqs. are parabolic, that is, they describe an initial value problem in t-direction. Solution can be obtained by integration in t-direction, starting from given initial conditions.
  $\Rightarrow$ downstream conditions have no upstream influence
  - The eqs. become invalid near separation points!
- The continuity equation is usually eliminated by introducing a stream function ψ, such that $u_t = \frac{\partial \psi}{\partial n}$ and $u_n = -\frac{\partial \psi}{\partial t}$.

Consequences of B.L.: (S&G, pp. 150 - 152) (Moran, pp. 199 - 202)

A) Skin friction:

$$\tau_w = \mu \frac{\partial u_t}{\partial n}\Big|_{n=0}$$

$\Rightarrow$ Friction drag force:

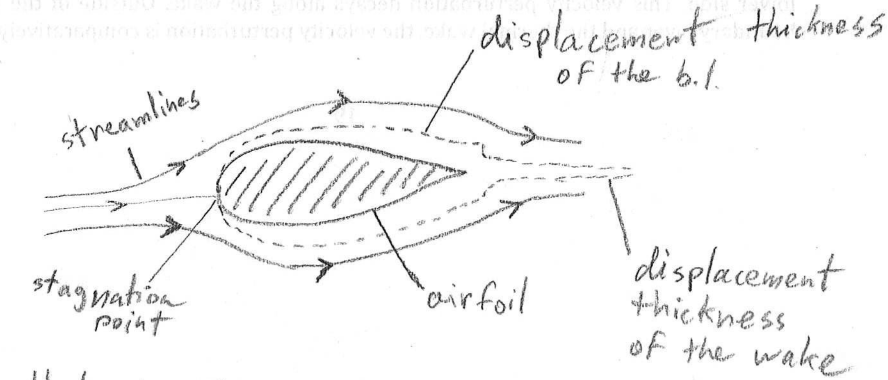
$$\vec{F}_\tau = \oint \tau_w \vec{t}\, ds$$

(Moran, p. 199)

(S&G, pp. 154-155)

B) Displacement thickness & Form drag

The b.l. modifies the outer flow (second-order effect). For the outer flow, the b.l. increases the apparent "displacement thickness" of the airfoil:



Note that the displacement thickness does not vanish at the trailing edge. → no stagnation pressure

(7)

Form drag

The displacement thickness of the b.l. does not vanish at the trailing edge, but it continues to the wake. Therefore, the pressure at the trailing edge is lower than the stagnation pressure.

⇒ the difference between the stagnation pressure at the stagnation point and the lower pressure at the trailing edge creates a "form drag": $F_F = \oint p \frac{\vec{u}_\infty}{|\vec{u}_\infty|} d\vec{S}$,

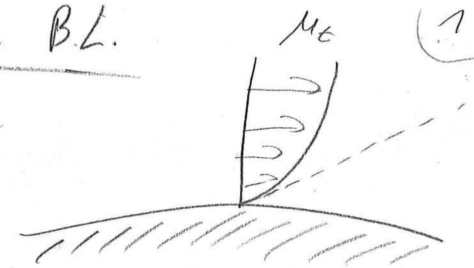
that is, a non-zero pressure force in the direction of the incoming flow. In other words, the total pressure force is no longer perpendicular to the incoming flow.

c) Flow separation: (Moran, pp. 222–225)
(S&G, pp. 150–151)

- The boundary layer can separate in regions of "adverse pressure gradient", where $\frac{\partial p}{\partial t} > 0 \Rightarrow \frac{\partial U_e}{\partial t} < 0$, that is, when the outer flow decelerates (typically upper side of the airfoil).
- The classical boundary layer equations break down near the separation.
- Predicting and computing flow separation is difficult (advanced courses). → Requires interaction between b.l. and outer flow
- Flow separation reduces lift and increases form drag
- Separation often accompanied by Histeresis

⇒ Stall – sudden massive separation
– loss of lift and massive increase of form drag

Consequences of the B.L.



~~DRAG~~

(A) Skin friction

$$\tau_w = \mu \frac{\partial u_t}{\partial n}\Big|_{n=0}$$

$$\vec{F}_\tau = \oint_S \tau_w \cdot \vec{t}\, ds$$

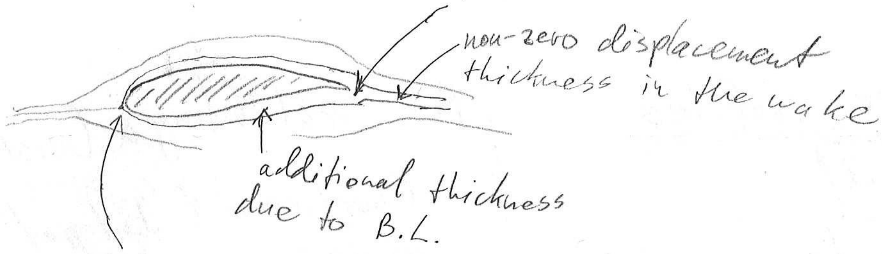
(C) Flow separation

→ stall !

(form drag)

(B) Displacement thickness

For the outer flow, B.L. acts as additional thickness of the airfoil (~~flat plate~~)



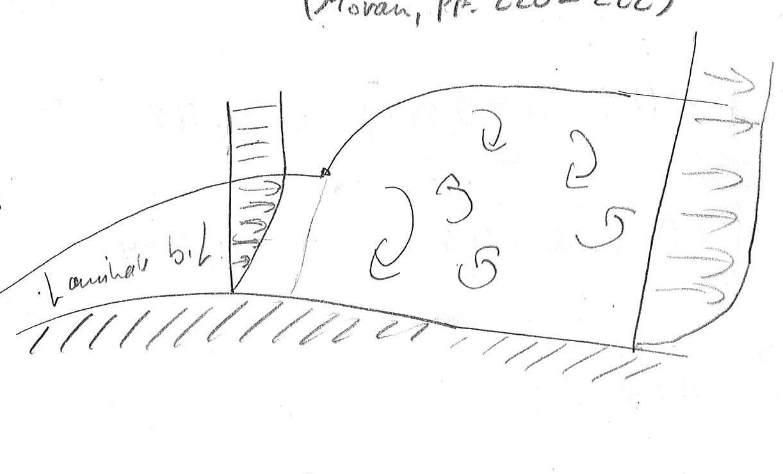
$$p - p_\infty = \frac{1}{2} \rho_\infty U_\infty^2$$

Short comings:

D) Transition to turbulence

(Moran, pp. 220–222)

- at some point, chaotic vortices start to grow in the B.L.
  → Larger τ_w



→ the ~~nd~~ dimensionless) position of the transition point depends on Re and on the pressure distribution

⇒ drag reduction → delay the transition to turbulence

Laminar boundary layer:

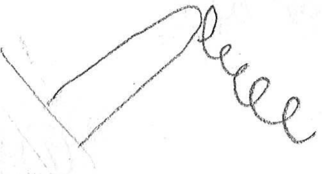
- for typical (thick) NACA 4-digit airfoils:
  - laminar b.l. predicts separation, even at $\alpha = 0°$
  - in reality - transition to turbulence, soon after the point of minimum pressure
  - separation / transition point not sensitive to Re!
    Re occurs only in $C_f(S)$

• **Laminar-flow airfoils** - for gliders
  - thin
  - transition postponed by shifting point of max. thickness downstream
  - restricted to very small α

• **Drag force**

- friction drag: $\oint c_f \cdot \vec{t}\, ds \cdot \frac{\vec{u}_\infty}{|u_\infty|}$
- form drag: displacement of streamlines by B.L.
  $\Rightarrow$ reduced pressure at trailing edge

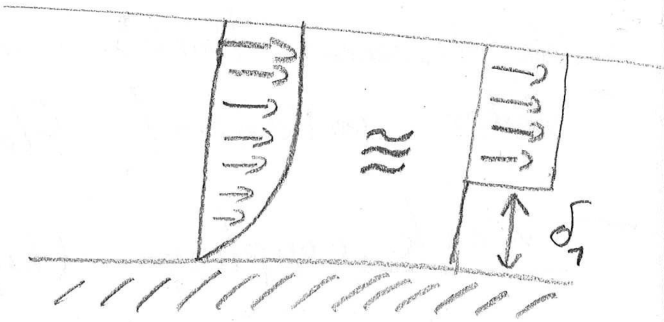
Induced drag:
- wing-tip vortices
- 3D effect



$\Rightarrow$ Higher-order B.L. theory

· Displacement thickness:

$$\delta_1 = \frac{1}{U_e} \int_0^\infty (U_e - u)\, dy$$



(δ^* in Moran)

· Momentum thickness:

$$\delta_2 = \frac{1}{U_e^2} \int_0^\infty u(U_e - u)\, dy$$

(θ in Moran)

⇒ common shape factors: $Re_2 = \frac{\rho U_e \delta_2}{\mu}$

(Re_θ), p. 206 of Moran

$H = \frac{\delta_1}{\delta_2}$ (p. 204 of Moran)

Higher-order boundary-layer theory:

- displacement thickness δ_1 of the B.L. modifies the outer potential flow

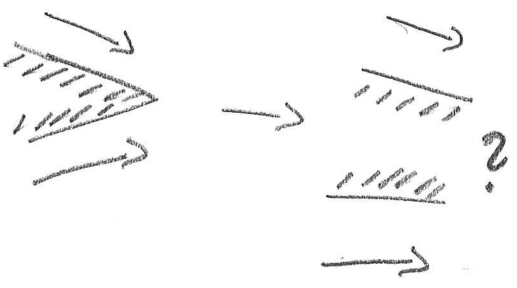
- weak coupling (higher order B.L.T.):

  1) Compute outer flow without B.L.
  2) Compute B.L. $\rightarrow \delta_1$
  3) Correct outer flow using δ_1

     a) modify body shape by δ_1

     b) prescribe normal displacement velocity at wall: $U_n\Big|_{\frac{y}{L}\to 0} = \frac{d}{dx}(U_e\,\delta_1)$

⚠ Trailing edge:



- displacement thickness of the wake $\Rightarrow$ no stagnation pressure

- Singularity $\Rightarrow$ strong interaction

  - outer flow & B.L. solved simultaneously

  $\Rightarrow$ Tripple-Deck theory $\Rightarrow$ c_D, c_f correction