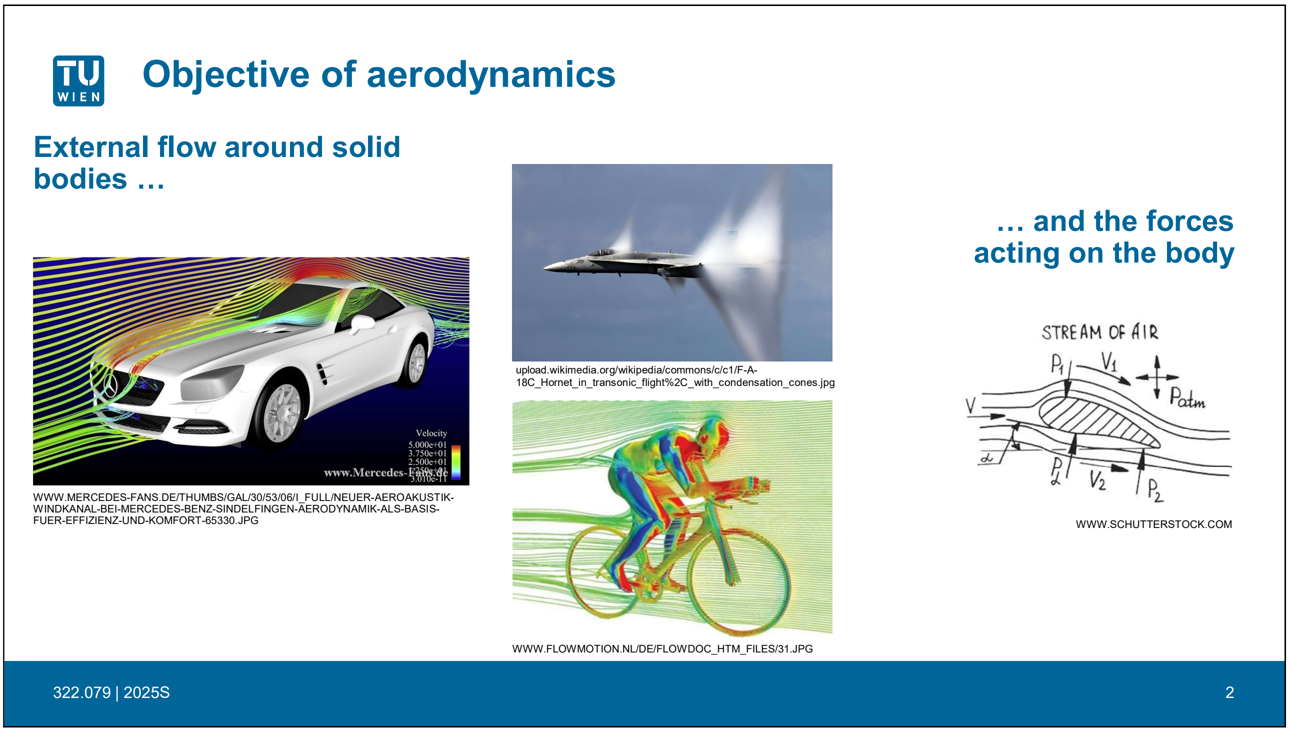
2 Introduction



The objective of aerodynamics is to compute the flow of air over objects and the forces by which the flow acts onto the objects. In this course, we will focus on the aerodynamics of airfoils (the main part of a classical wing), but the methods are generalizable to other geometries. The sketch on the right side of the slide illustrates the flow over an airfoil (wing profile). The force acting on the body arises by two mechanisms: pressure (acting perpendicular to the surface) and friction (acting tangentially to the surface). The net force, obtained by integration of the stresses over the surface of the airfoil, is formally divided into two components – one perpendicular and one parallel to the direction of the incoming flow. The former is called “lift”, and the latter is called “drag”.

The difficulty of computing the flow over a wing profile at practically relevant conditions is illustrated, e.g., by M. Hosseini et al. (2015). Close to the surface of the wing there is a thin boundary layer and behind the wing there is a slender wake. Typically, the wake and most of the boundary layer are turbulent. This means they contain many vortices of various sizes, that are distributed chaotically in space and time (see Figure 1). If one wants to compute the flow by solving the governing equations of motion numerically (using a digital computer), it is necessary to compute all these vortices. This way, to compute the flow over a wing profile of a small glider already requires 89 days on a supercomputer using 16 384 CPU cores (S. M. Hosseini et al., 2016). For a larger aircraft the computational requirements would be even larger. Therefore, it is clearly impractical to solve the governing equations directly, when many different geometries are to be tested in order to find an optimal design.

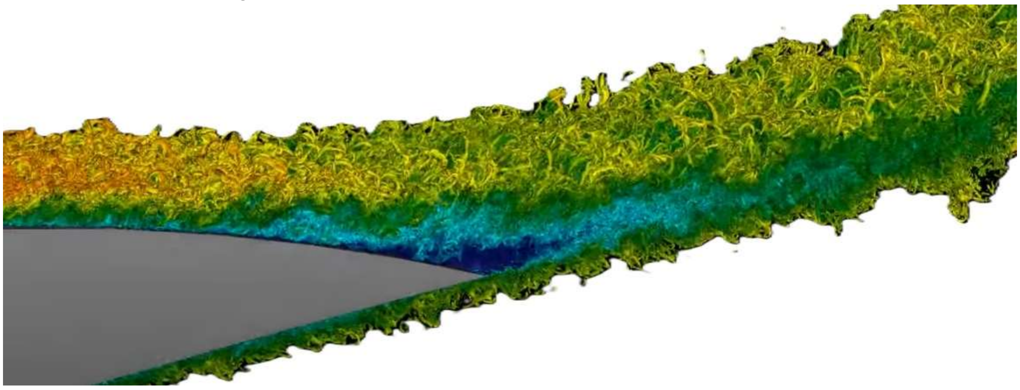


Figure 1: Vortices near the trailing edge of a small glider wing. Taken from M. Hosseini et al. (2015).

Methods of computation

Flow described by equations of motion:

$$\dot{m} = 0, \qquad \boldsymbol{F} = m\boldsymbol{a}$$

Direct Numerical Simulation (DNS)

- Solve EoM at discrete points in space (computational grid) → large system of equations
- Sufficient resolution to capture all features of the flow
- Thin section of a glider wing takes ~10^9 grid points → 89 days on 16 384 CPU cores[1]

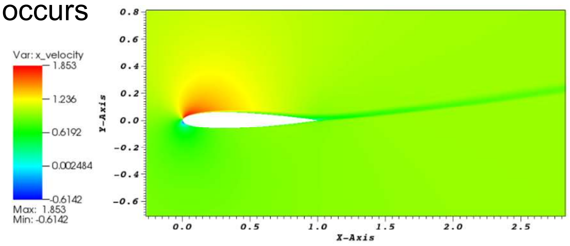
Turbulence modelling

Large Eddy Simulation (LES)

- Filter-out smallest vortices and model their effect
- ~10^8 grid points[2]

Reynolds Averaged Navier-Stokes (RANS)

- Time-averaged flow
- Effect of all turbulent fluctuations modeled by additional equations
- Lower resolution → computable on PC
- Unreliable when flow separation (stall) occurs



Min, M. et al. (2019) Nek5000 enhancements for faster running analysis.

[1] Hossein, S.M. et al. (2016) *Int. J. Heat Fluid Flow*, pp. 117-128

[2] Vinuesa, R. et al. (2018) *Int. J. Heat Fluid Flow*, pp. 86-99

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A possible remedy for the large computational costs is to model the overall effect of the vortices on the flow without computing them. This can be achieved by supplementing the equations of motion by modelled terms and additional model equations. Two main approaches for modelling the effect of turbulent vortices on the flow exist. These are the Large Eddy Simulation (LES) and the Reynolds Averaged Navier-Stokes equations (RANS). In the former case, vortices smaller than a certain threshold are filtered out, and their effect on the larger structures in the flow is modelled. The solution of such modelled equations contains only the larger vortices. Thus, it can be computed with lower resolution (in space and time) and lower computational cost. It turns out that the effect of the smallest vortices is rather easy to model. Therefore, LES is still a very accurate method. Although its numerical cost is lower than solving the original equations of motion directly, it is usually still too large for practical applications.

RANS is a much cheaper computational method. It computes the statistically steady flow averaged over a sufficiently long time, such that the result is independent of the time window selected for averaging. The solution of the Reynolds averaged equations is steady and does not contain any of the chaotic turbulent vortices. Thus, it can be computed with much lower computational costs on a single personal computer in a reasonable time. However, the massive reduction of the computational effort comes at the expense of low accuracy and reliability. Turbulence models for RANS namely rely on a number of empirical constants that are not universal. The values of these constants are typically tuned only for simple flow conditions. Therefore, RANS computations usually cannot predict reliably phenomena like flow separation or transition from laminar to turbulent flow.

The methods mentioned so far (DNS, LES, RANS) describe all regions of the flow with a single set of equations. These equations consist of fundamental physical laws, namely the conservation of mass, conservation of linear momentum, and conservation of energy. In the case of LES and RANS, these physical laws are supplemented with artificial model equations. In either case, the governing equations remain difficult to solve. They cannot be simplified much if they are required to describe the entire flow, because every term of these equations is potentially important in some regions of the flow.

Methods of computation

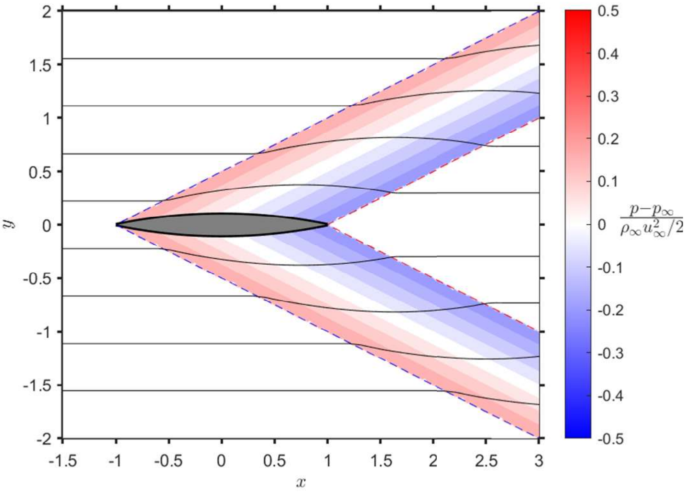
Boundary-Layer theory

- Divide flow into boundary layer and outer flow
- Lift determined by outer flow
- Drag caused by boundary layer
- Flow in each region approximated with simplified equations
- Outer flow is potential:

  $u = \phi'_x, \qquad v = \phi'_y$

  → Potential Flow Theory

Panel method



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In order to find a more efficient computational method, one can take advantage of empirical observations which reveal the general structure of the flow and search for a method that is tailored specifically to the problems of aerodynamics. From the video of M. Hosseini et al. (2015) we observe that the turbulent vortices appear only inside the thin boundary layers and the wake. Outside of these shear layers, the flow is quite simple. Furthermore, it turns out that the time-averaged boundary layers have a somewhat universal structure and can be described with a small number of parameters. These observations motivate the Boundary-Layer Theory.

The idea is to divide the flow into two regions. One region consists of the shear layers, that is, the boundary layers and the wake. The second region is the outer flow outside of these shear layers. Then, the flow in each region is to be described with a different set of equations. This way, each set of equations describing a different region of the flow can be simplified dramatically.

The boundary layer theory takes advantage of the fact that, in typical conditions studied by aerodynamics, the boundary layers are very thin as compared to the size of the body in the flow. Therefore, it turns out that the pressure is to a good approximation constant across the thickness of the boundary layers (unless buoyancy affects the flow, which can happen when the body is heated or cooled). The pressure is imposed onto the boundary layer from the outer flow. This has several consequences. First, the pressure distribution over the surface of the body can be computed by solving only the simplified equations describing the outer flow. This result is sufficient to obtain the lift force, without even solving the equations describing the boundary layers. The solution of the boundary layer equations then leads to a small correction of the lift force.

A second consequence of the fact that the pressure is determined by the outer flow is a simplification of the equations describing the boundary layers. The simplified boundary-layer equations are parabolic, while the original equations of motion were elliptic (we will discuss what this means in more detail). In other words, the boundary layer equations become an initial value problem, that can be solved by integration starting from initial conditions. The derivation and solution of the boundary layer equations is covered in a dedicated course, VO 322.029 “Boundary layer theory”. In this course we will restrict ourselves to the numerical solution of the boundary layer equations.

The equations governing the outer flow will be derived in the first part of this course. We will see that, in many cases, the equations are as simple as Laplace equation or wave equation. For the flow over simple shapes, these equations can be solved analytically. As an example, the figure on the right side of the slide above shown the analytical solution for the pressure field and streamlines around a thin profile moving through a fluid with zero lift at supersonic speed. For general shapes, the equations can be solved numerically. We will cover the Panel Method, which is a numerical method that obtains the solution for the outer flow using computational grid points that are distributed only along the surface of the body.

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Focus of this course

- Aerodynamics of airfoils
- Approximate computations with low computational effort
  - → Simplified equations
  - → Combination of analytical and numerical tools

Content

- Potential flow theory
  - → Analytical flow around basic shapes
  - → Computation of lift
- Panel method
- Laminar and turbulent boundary layers
  - → Computation of drag

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In this course we will learn how to compute the aerodynamics of an airfoil with low computational effort. First, we will cover the potential flow theory, that describes the outer flow and determines the lift force. After deriving the equations for the outer flow we will cover their exact analytical solutions for some simple shapes. Next, we will learn the panel method for computing the outer flow numerically, and we will implement the method in a computer program. Finally, we will see how to compute the boundary layers in order to obtain the drag force of the airfoil.

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