1 Hands on

Vectors

- Create a row vector a using command `a = [ 1 2 3 4]`
- Create a column vector b using command `b = [1;2;3;4]`
- You can transpose a vector using `tranpose(b)` or `b'`
- Scalar product can be done with `a*b`
- Term by term product can be done with `a.*b`
- Term by term dividing can be done with `a./b`
- Compute the norm-2 of a vector
- what does `a(2:end)-a(1:end-1)` does ?

Matrices

- Type `A = [1 2;3 4]` to create the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

- One can change to 17 the element on the second row, third column of that matrix with `A(2,3)=17`
- Use the function `eye` to create an identity matrix
- Look at the functions `diag`,`ones` , and create an identity matrix by using these two functions.
- By adding three matrices created with the diag function, create a tridiagonal matrix
- Use the command `inv` to invert a matrix
- To solve the linear problem $Ax = b$ with matlab, one option is `b=A\b` (backslash is important : `A\b`≠`b/A`).

Solving your first numerical problem

We want to solve the stationary 1D heat equation on the domain $\Omega = [0, 1]$ with a source term $f(x) = sin(2\pi x)$.

The problem we want to solve is : find T such that

$$\begin{cases} \dfrac{\partial^2 T}{\partial x^2} = f \text{ on } \Omega \\ T(0) = T(1) = 0 \end{cases} \tag{1}$$

- Create a script that you will call `heat1D.m`.
- Create the space variables : `N=11;dx=1/(N-1) ; x=[0:dx:1]';` This will create a column vector with the spatial coordinates.
- Create the source term with `f = sin(2*pi*x);`
- Create the following tridiagonal matrix

$$A = \begin{pmatrix} 1 & 0 & \cdots & & \cdots & 0 \\ \frac{1}{\Delta x^2} & \frac{-2}{\Delta x^2} & \frac{1}{\Delta x^2} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & \frac{1}{\Delta x^2} & \frac{-2}{\Delta x^2} & \frac{1}{\Delta x^2} \\ 0 & \cdots & & \cdots & 0 & 1 \end{pmatrix}$$

  with the command lines

```
A = eye(N,N);
for i=2:N-1
   A(i,i-1)=  1/dx^2;
   A(i,i)  = -2/dx^2;
   A(i,i+1)=  1/dx^2;
end
```

- Solve the linear problem $AT = f$ as indicated in the previous section.
- Using the commands `tic` and `toc`, find the time needed to solve the linear problem for different `N`, e.g. N=100, N=1000, N=10000.
- Change `A = eye(N,N);` into `A = speye(N,N);` . How much time does it take to solve the linear problem ? What are the advantages of a sparse matrix ?
- One can plot the solution with `plot(x,T)`, if `T` is the solution vector.
- *Bonus :* Find an other way to build the matrix, using `diag` or `spdiag`.