Fundamentals of Numerical Thermo-Fluid Dynamics 322.061

Exercise 5: Explicit Schemes

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In this exercise we propose to use different spatial and temporal schemes to solve the following 1^{st} order problem in 1D.

$$\begin{cases} \partial_t u = c\,\partial_x u \quad \text{, with } t \in \mathbb{R}^+, x \in [0,1], c \in \mathbb{R}^* \\ u(t=0,x) = e^{-\frac{(x-0.5)^2}{0.02}} \\ u(t,x=0) = u(t,x=1) \end{cases} \tag{1}$$

Note: for every question of this exercise, one needs to create a function that solves one iteration. This function will then be called in a provided code template.

5.1) Implement a function creating the matrix associated to the Explicit Euler scheme in time and backward in space, with periodic boundary conditions.

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} = c\frac{U_i^n - U_{i-1}^n}{\Delta x} \tag{2}$$

5.2) Implement a function creating the matrix associated to the Explicit Euler scheme in time and forward in space, with periodic boundary conditions.

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} = c\frac{U_{i+1}^n - U_i^n}{\Delta x} \tag{3}$$

5.3) Implement a function creating the matrix associated to the Explicit Euler scheme in time and centered in space, with periodic boundary conditions.

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} = c\frac{U_{i+1}^n - U_{i-1}^n}{2\Delta x} \tag{4}$$

5.4) Implement a function creating the matrix associated to the Leap-Frog scheme, with periodic boundary conditions.

$$\frac{U_i^{n+1} - U_i^{n-1}}{2\Delta t} = c\frac{U_{i+1}^n - U_{i-1}^n}{2\Delta x} \tag{5}$$

Caution : one has to store one more time step. Therefore the template has to be changed.

5.5) Compare the stability condition of all this different schemes (*cf.* exercise 3)