

The Runge-Kutta methods are iterative ways to calculate the solution of a differential equation. Starting from an initial condition, they calculate the solution forward step by step. The second-order formula (RK2) is:

$$\begin{aligned}k_1 &= hf(x_n, y_n), \\k_2 &= hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right), \\y_{n+1} &= y_n + k_2 + O(h^3).\end{aligned}\tag{1}$$

For the third-order formula, it holds:

$$\begin{aligned}k_1 &= hf(x_n, y_n), \\k_2 &= hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right), \\k_3 &= hf(x_n + h, y_n - k_1 + 2k_2), \\y_{n+1} &= y_n + \frac{1}{6}(k_1 + 4k_2 + k_3) + O(h^4);\end{aligned}\tag{2}$$

and the forth-order formula (RK4) is:

$$\begin{aligned}k_1 &= hf(x_n, y_n), \\k_2 &= hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right), \\k_3 &= hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right), \\k_4 &= hf(x_n + h, y_n + k_3), \\y_{n+1} &= y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) + O(h^5).\end{aligned}\tag{3}$$

This method is reasonably simple and robust and is a good general candidate for numerical solution of differential equations. It should be noted that the methods explained here are all explicit.