Fundamentals of Numerical Thermo-Fluid Dynamics 322.061

Exercise 6: Implicit Schemes

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In this exercise we propose to use different spatial and temporal schemes to solve the following 2[nd] order problem in 1D.

$$\begin{cases} \partial_t T = D\,\partial_{xx} T \quad ,\text{with } t \in \mathbb{R}^+, x \in [0,1], D \in \mathbb{R}^+ \\ T(t=0,x) = 1 - x + e^{-\frac{(x-0.5)^2}{0.02}} \\ T(t,x=0) = 1 \quad \dfrac{\partial}{\partial x} T(t,x=1) = 0 \end{cases} \tag{1}$$

Note: the code template provided for exercise 5 can be adapted/optimized.

5.1) Solve numerically the problem using the implicit Euler Scheme.

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = D\frac{T_{i-1}^{n+1} - 2T_i^{n+1} + T_{i+1}^{n+1}}{\Delta x^2} \tag{2}$$

5.2) Solve numerically the problem using the Crank-Nicholson scheme.

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \frac{D}{2}\frac{T_{i-1}^{n+1} - 2T_i^{n+1} + T_{i+1}^{n+1}}{\Delta x^2} + \frac{D}{2}\frac{T_{i-1}^{n} - 2T_i^{n} + T_{i+1}^{n}}{\Delta x^2} \tag{3}$$

In the following question, we add a convection term, the problem becomes

$$\begin{cases} \partial_t T = c\partial_x T + D\,\partial_{xx} T \quad ,\text{with } t \in \mathbb{R}^+, x \in [0,1], D \in \mathbb{R}^+, c \in \mathbb{R}^* \\ T(t=0,x) = 1 - x + e^{-\frac{(x-0.5)^2}{0.02}} \\ T(t,x=0) = 1 \quad \dfrac{\partial}{\partial x} T(t,x=1) = 0 \end{cases} \tag{4}$$

5.3) Solve numerically the problem using the Implicit Euler scheme.

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = c\frac{T_{i+1}^{n+1} - T_{i-1}^{n+1}}{2\Delta x} + D\frac{T_{i-1}^{n+1} - 2T_i^{n+1} + T_{i+1}^{n+1}}{\Delta x^2} \tag{5}$$

5.4) Knowing that implicit schemes are often more stable than the explicit ones, why would you then prefer using Explicit schemes over Implicit schemes when one wants a high resolution in time ?