Fundamentals of Numerical Thermo-Fluid Dynamics 322.061
Examples for home preparation

Exercise 5: Implicit time-stepping schemes

To be presented on June 19, 2019

5.1 Solve numerically the one-dimensional advection of diffusive scalar

$$\begin{cases} \partial_t T = -u\partial_x T + D\ \partial_{xx} T \quad , \text{on } \Omega = [0,1] \\ T(t=0,x) = 1 - x + e^{-\frac{(x-0.5)^2}{0.02}} \\ T(t,x=0) = 1 \quad \frac{\partial}{\partial x} T(t,x=1) = 0 \end{cases} \tag{1}$$

with diffusivity $D = 10^{-2}$ by a flow of constant velocity $u = 1$. First discretize the problem in space on a grid with uniform spacing $\Delta x = 10^{-2}$ using second order central differencing, to form an initial value problem

$$\frac{\partial T_j}{\partial t} = A_{jk} T_k \tag{2}$$

Then solve this initial value problem in MatLab using implicit Euler scheme with time-step $\Delta t = 10^{-2}$. Show the transient transport of T until steady state.

5.2 Solve the problem from previous question with Crank-Nicolson scheme and compare to the result of implicit Euler scheme. The difference between the two results can be used as an estimate of the numerical error of time integration. Reduce the length of time step to obtain sufficiently small error.

5.3 Now assume that we want to impose a zero-flux on both boundaries, such that the total amount of the quantity T inside of the domain Ω is conserved. This can be achieved by applying Robin condition on both boundaries

$$(\vec{u} \cdot \vec{n})T - D\frac{\partial T}{\partial \vec{n}} = 0 \quad \text{at } x = 0 \text{ and } x = 1$$

where $\vec{n}$ is the outward-pointing normal to the domain boundary. Apply these boundary conditions to the problem (1) and solve it with both implicit Euler and Crank-Nicolson scheme. Compare the results.

5.4 Consider a fully developed two-dimensional laminar channel flow between two infinite horizontal plates with distance $y_T - y_B = 1$. At $x_L = 0$ the flow enters a test section of length $x_R - x_L = 10$ where the temperature of the plates jumps impulsively from the temperature of the fluid $T_T = T_B = 0$ at $t < 0$ to a different temperature $T_T = T_B = 1$ at $t > 0$. The temperature distribution of the fluid in the heated section is then described by

$$\begin{cases} \partial_t T = -u\partial_x T + \kappa(\partial_{xx} T + \partial_{yy} T) \quad , \text{on } \Omega = [0, 10] \times [0, 1] \\ T(t = 0, x, y) = 0 \\ T(t, x = 0, y) = 0 \quad \dfrac{\partial}{\partial x} T(t, x = 10, y) = 0 \\ T(t, x, y = 0) = 1 \quad T(t, x, y = 1) = 1 \end{cases} \tag{3}$$

Compute the evolution of the thermal boundary layer in the test section, using the template `Template_5_4.m` Apply second order central differences in space and implicit Euler scheme in time.