Supersonic wind tunnel

The supersonic flow over an inclined plate, around cones with different cone angles and around blunt bodies is visualized using a Schlieren method. Regarding the flow over an inclined plate, measurements of the shock angle are compared with available theoretical predictions. The properties of the fluid before and after the shock are determined from the measured data and with the aid of theoretical results.

1 Wind tunnel

An atmospheric inlet wind tunnel is operated by drawing air from the atmosphere through a Laval nozzle and the test section into a vacuum tank, see Fig. 1. Because the temperature of the air decreases when it is accelerated to sonic speeds (cf. with the properties of isentropic flow, Table 1), humidity must be removed from the air before it enters the wind tunnel. The Laval nozzle and the test section of the wind tunnel are designed in such a way to produce a fairly two-dimensional flow. The properties of the flow in the test section can be calculated by assuming isentropic flow. The flow does not remain isentropic across shocks.

The flow is visualized by the Schlieren method, see Fig. 2. A point (or line) source of light is collimated to a parallel beam, passes through the test section and is focused on the knife edge. The knife edge cuts off part of the light. When a ray of light passes a location in the test section with a gradient of the refractive index normal to the orientation of the edge, the light is deflected and the brightness of the image changes at that point. The index of refraction of a gas depends on its density, hence, the gray values in a Schlieren image correspond to density gradients normal to the knife edge.

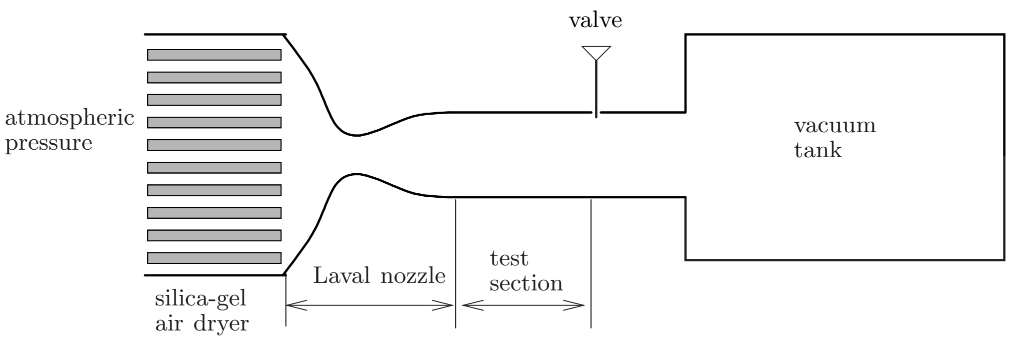

Figure 1: Atmospheric inlet wind tunnel.

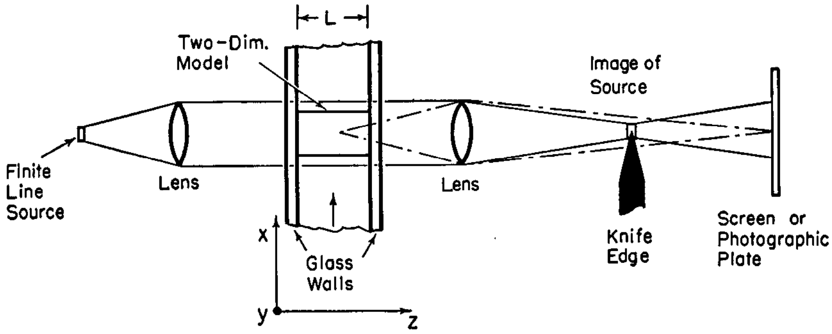

Figure 2: Schlieren system. (Shapiro, 1953, Fig. 3.12).

1.1 Problems

1.1 Measure the area ratio A/A^* for the test section and determine the properties of the flow in the test section, i.e., M_1, p_1, ρ_1 and T_1.

1.2 Give the maximum back pressure, i.e., the pressure in the vacuum tank, that is necessary to initiate supersonic flow. To calculate the back pressure, it can be assumed that (i) the flow is isentropic from the atmosphere through the Laval nozzle and the test section, (ii) a normal shock is located at the end of the test section and (iii) behind the shock, the fluid comes isentropically to rest.

1.3+ The vacuum tank has a capacity of 50 m^3 and can be evacuated to 3 mbar. Calculate the operation time of the wind tunnel. Air can be assumed to be a perfect gas with the specific gas constant $R = 287$ J/kgK.

2 Experiments

Various bodies are placed inside the test section. The flow is established for a short period of time by opening the valve. The Schlieren image is recorded with a CCD-camera, to facilitate observation and measurements. Sketch the flow fields accurately. Clearly indicate shocks and expansion fans. State your observations and interprete your results.

2.1 A plate is inserted into the flow. The flow around the plate is visualized for several inclination angles between 0° and approx. 40°. Measure the inclination angle of the plate θ and the angle of the oblique shock. Compare the measured values with theory.

Use Figs. 7 and 8 to determine the flow properties behind the shock, M_2, p_2 and ρ_2. Also, with the aid of Table 1, determine the change of the stagnation pressure.

2.2 Find the inclination angle at which the shock detaches from the plate. Compare the value with theory.

2.3 Sketch the flow around cones with different cone angles and around bodies with a blunt nose. Can you say something about the drag of the bodies?

Appendix: Important results from gas dynamics

A.1 Area-velocity relation

The governing equations for the steady, one-dimensional, isentropic flow in the absence of body forces are given by,

$$\dot{m} = \rho A V = const., \tag{1}$$

$$\mathrm{d}p + \rho V \mathrm{d}V = 0, \tag{2}$$

$$h + V^2/2 = const., \tag{3}$$

$$s = const., \tag{4}$$

where A is the cross-sectional area of the duct, V is the velocity, h denotes the enthalpy and s the entropy of the fluid, respectively. The speed of sound a is given by

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s . \tag{5}$$

For $s = const.$, the partial derivatives in eq. (5) can be replaced by the ordinary derivatives, $a^2 = \mathrm{d}p/\mathrm{d}\rho$, which together with eqs. (1) and (2) yields the area-velocity relation, (Zucrow and Hoffman, 1976, p. 166), (Anderson, 1982, p.124),

$$\frac{\mathrm{d}A}{A} = (M^2 - 1)\frac{\mathrm{d}V}{V}. \tag{6}$$

Here M is the Mach number, $M = V/a$. The area-velocity relation states that a converging flow passage acts as a nozzle for subsonic flow, but as a diffusor for supersonic flow. The reverse is valid for a diverging passage, see Fig. 3.

For a perfect gas,

$$p = \rho R T, \qquad h = c_p T, \tag{7}$$

the speed of sound is given by $a = \sqrt{\gamma R T}$.

A.2 Laval nozzle

To obtain supersonic velocity, a Laval nozzle, or convergent-divergent nozzle, is used, see Fig. 4. In the throat of the Laval nozzle the fluid reaches the critical condition, i.e., the fluid velocity is equal to the speed of sound, $V_\mathrm{t} = a_\mathrm{t}$, where the index t refers to the state in the throat. Hence, by definition, $a_\mathrm{t} = a^*$, $M_\mathrm{t} = M^* = 1$ and $\dot{m}_\mathrm{t} = \dot{m}^*$.

A.3 Stagnation and critical properties

The stagnation properties of a fluid are defined as the properties which the fluid attains when it is brought to rest in an isentropic process. Hence, the stagnation properties are uniquely determined and serve as a useful reference frame for the state of the fluid. In an isentropic flow, the stagnation properties remain constant.

The ratio of the local properties to the stagnation properties for a perfect gas are tabulated in Table 1. They are given by (Shapiro, 1953, p. 83), (Zucrow and Hoffman, 1976, p. 173),

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2, \quad \frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}, \quad \frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma-1}}. \tag{8}$$

Useful parameters are the ratio of the area of the duct to the critical area, which usually is the throat area of the Laval nozzle, and the critical Mach number, $M^* = V/a^*$, (Zucrow and Hoffman, 1976, p. 175),

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma - 1}{2} M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}, \qquad M^{*2} = \frac{(\gamma+1)M^2}{2 + (\gamma-1)M^2}. \tag{9}$$

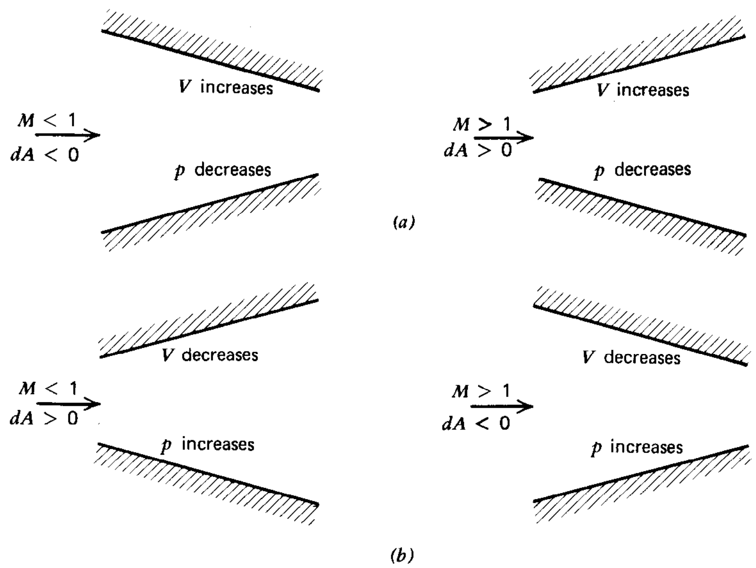

Figure 3: Effect of area change on steady one-dimensional isentropic flow. (a) Nozzle flow. (b) Diffusor flow. (Zucrow and Hoffman, 1976, Fig. 4.4).

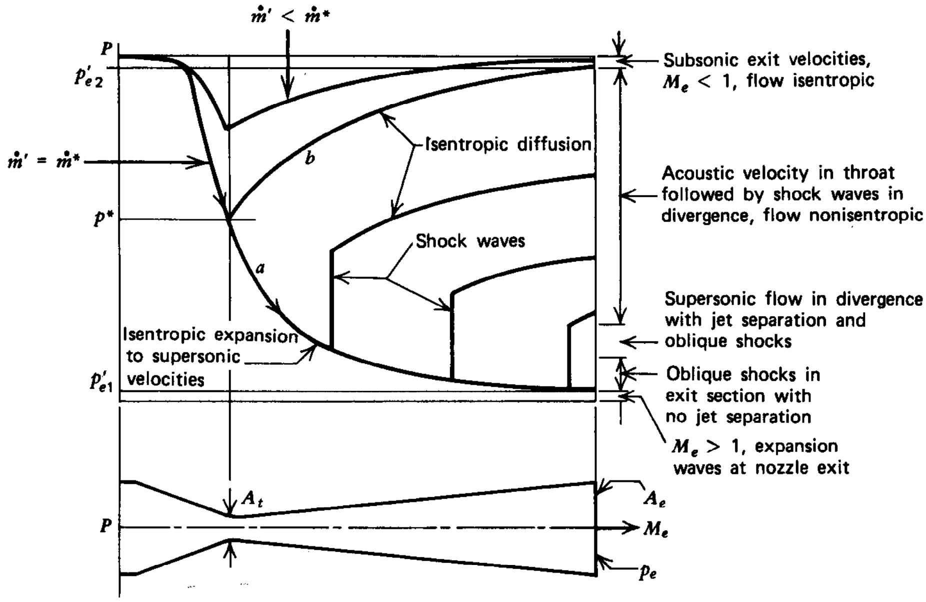

Figure 4: Static pressure distribution in a Laval nozzle. In this Figure, the stagnation pressure is denoted by P. (Zucrow and Hoffman, 1976, Fig. 4.23).

A.4 Normal shock

Across a normal shock, the balances of mass, momentum and energy are given by (Zucrow and Hoffman, 1976, pp. 336)

$$\rho_1 V_1 = \rho_2 V_2, \tag{10}$$

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2, \tag{11}$$

$$h_1 + V_1^2/2 = h_2 + V_2^2/2, \tag{12}$$

where the indices 1 and 2 refer to the states upstream and downstream of the shock, respectively. For a perfect gas, eqs. (10) to (12) yield the following relations (Zucrow and Hoffman, 1976, p. 342),

$$M_2^2 = \left(M_1^2 + \frac{2}{\gamma-1}\right) \Big/ \left(\frac{2\gamma}{\gamma-1} M_1^2 - 1\right), \tag{13}$$

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1}, \qquad \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}, \tag{14}$$

$$\frac{p_{0,2}}{p_{0,1}} = \left[\left(\frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)M_1^2}\right)^{\gamma} \left(\frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1}\right)\right]^{-1/(\gamma-1)}, \tag{15}$$

which are tabulated in Table 2.

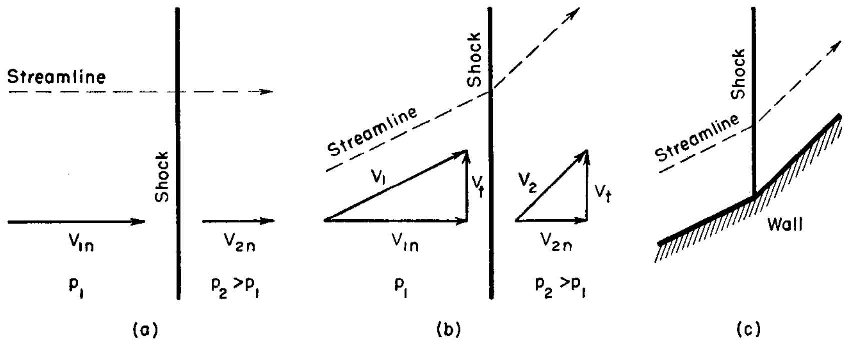

Figure 5: Transformation of normal shock to oblique shock. (a) Stationary observer sees normal shock. (b) Observer moving along shock line sees oblique shock. (c) Flow of (b) interpreted as pattern produced near concave corner in wall. (Shapiro, 1953, Fig. 16.1).

Equation (13) yields

$$M_1^* M_2^* = 1, \tag{16}$$

which shows that a normal shock always leads to subsonic flow. For M_1 larger than approx. 1.3, a normal shock considerably increases the entropy of the flow. Note that across a normal shock the stagnation enthalpy, $h_0 = h + V^2/2$, remains constant, hence for a perfect gas the stagnation temperature also remains constant.

A.5 Oblique shock

In an oblique shock, the velocity tangent to the shock line remains constant across the shock, while the normal velocity changes according to the normal shock relations, cf. Fig. 5. The shock relations are presented in graphical form in Figs. 7 and 8. The flow behind an oblique shock can be supersonic. The inclination of an oblique shock is always larger than the inclination of the Mach line. If the inclination of the shock is only slightly larger than the Mach line, the Mach number of the velocity component normal to the shock is not much larger than unity and the entropy increase across the shock is small.

A.6 Prandtl-Meyer expansion

The flow in a centered expansion fan, or Prandtl-Meyer expansion, is isentropic. The fluid expands continuously from the state before the expansion fan to the state behind the expansion fan. The flow properties propagate along Mach lines correponding to the local state of the fluid and remain constant along these characteristics, see Fig. 6.

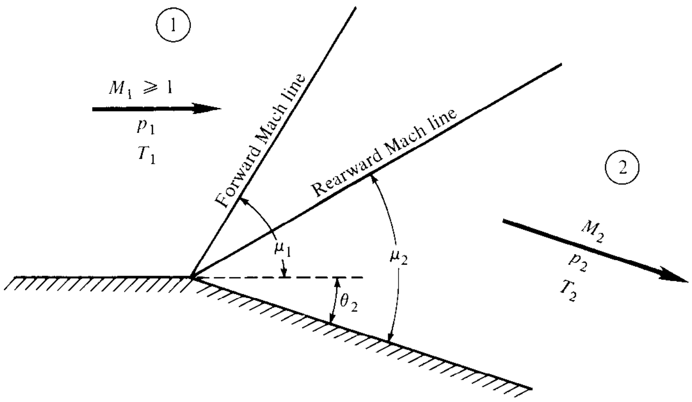

Figure 6: Prandtl-Meyer expansion. (Anderson, 1982, Fig. 4.26).

References

Anderson, Jr., J. D. *Modern compressible flow.* (McGraw-Hill, 1982).

Liepmann, H. W. and Roshko, A. *Elements of gasdynamics.* (Wiley, 1957).

Shapiro, A. H. *The dynamics and thermodynamics of compressible fluid flow. Volume I.* (Ronald, New York, 1953).

Zucrow, M. J. and Hoffman, J. D. *Gas dynamics. Volume I.* (Wiley, 1976).

M	M^*	p/p_0	ρ/ρ_0	T/T_0	A/A^*
0	0	1	1	1	∞
0.2	0.2182	0.9725	0.9803	0.9921	2.964
0.4	0.4313	0.8956	0.9243	0.969	1.59
0.6	0.6348	0.784	0.8405	0.9328	1.188
0.8	0.8251	0.656	0.74	0.8865	1.038
1	1	0.5283	0.6339	0.8333	1
1.05	1.041	0.4979	0.6077	0.8193	1.002
1.1	1.081	0.4684	0.5817	0.8052	1.008
1.15	1.12	0.4398	0.5562	0.7908	1.017
1.2	1.158	0.4124	0.5311	0.7764	1.03
1.25	1.195	0.3861	0.5067	0.7619	1.047
1.3	1.231	0.3609	0.4829	0.7474	1.066
1.35	1.266	0.337	0.4598	0.7329	1.089
1.4	1.3	0.3142	0.4374	0.7184	1.115
1.45	1.333	0.2927	0.4158	0.704	1.144
1.5	1.365	0.2724	0.395	0.6897	1.176
1.55	1.395	0.2533	0.375	0.6754	1.212
1.6	1.425	0.2353	0.3557	0.6614	1.25
1.65	1.454	0.2184	0.3373	0.6475	1.292
1.7	1.482	0.2026	0.3197	0.6337	1.338
1.75	1.51	0.1878	0.3029	0.6202	1.386
1.8	1.536	0.174	0.2868	0.6068	1.439
1.85	1.561	0.1612	0.2715	0.5936	1.495
1.9	1.586	0.1492	0.257	0.5807	1.555
1.95	1.61	0.1381	0.2432	0.568	1.619
2	1.633	0.1278	0.23	0.5556	1.688
2.05	1.655	0.1182	0.2176	0.5433	1.76
2.1	1.677	0.1094	0.2058	0.5313	1.837
2.15	1.698	0.1011	0.1946	0.5196	1.919
2.2	1.718	0.09352	0.1841	0.5081	2.005
2.25	1.737	0.08648	0.174	0.4969	2.096
2.3	1.756	0.07997	0.1646	0.4859	2.193
2.35	1.775	0.07396	0.1556	0.4752	2.295
2.4	1.792	0.0684	0.1472	0.4647	2.403
2.5	1.826	0.05853	0.1317	0.4444	2.637
2.6	1.857	0.05012	0.1179	0.4252	2.896
2.8	1.914	0.03685	0.09463	0.3894	3.5
3	1.964	0.02722	0.07623	0.3571	4.235
3.5	2.064	0.01311	0.04523	0.2899	6.79
4	2.138	6.586E-3	0.02766	0.2381	10.72
4.5	2.194	3.455E-3	0.01745	0.198	16.56
5	2.236	1.89E-3	0.01134	0.1667	25
6	2.295	6.334E-4	5.194E-3	0.122	53.18
7	2.333	2.416E-4	2.609E-3	0.09259	104.1
8	2.359	1.024E-4	1.414E-3	0.07246	190.1
9	2.377	4.739E-5	8.15E-4	0.05814	327.2
10	2.39	2.356E-5	4.948E-4	0.04762	535.9

Table 1: Isentropic flow for a perfect gas, $\gamma = 1.4$.

M_1	M_2	$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2}$	$\frac{p_2}{p_1}$	$\frac{p_{0,2}}{p_{0,1}}$
1	1	1	1	1
1.05	0.9531	1.084	1.12	0.9999
1.1	0.9118	1.169	1.245	0.9989
1.15	0.875	1.255	1.376	0.9967
1.2	0.8422	1.342	1.513	0.9928
1.25	0.8126	1.429	1.656	0.9871
1.3	0.786	1.516	1.805	0.9794
1.35	0.7618	1.603	1.96	0.9697
1.4	0.7397	1.69	2.12	0.9582
1.45	0.7196	1.776	2.286	0.9448
1.5	0.7011	1.862	2.458	0.9298
1.55	0.6841	1.947	2.636	0.9132
1.6	0.6684	2.032	2.82	0.8952
1.65	0.654	2.115	3.01	0.876
1.7	0.6405	2.198	3.205	0.8557
1.75	0.6281	2.279	3.406	0.8346
1.8	0.6165	2.359	3.613	0.8127
1.85	0.6057	2.438	3.826	0.7902
1.9	0.5956	2.516	4.045	0.7674
1.95	0.5862	2.592	4.27	0.7442
2	0.5774	2.667	4.5	0.7209
2.05	0.5691	2.74	4.736	0.6975
2.1	0.5613	2.812	4.978	0.6742
2.15	0.554	2.882	5.226	0.6511
2.2	0.5471	2.951	5.48	0.6281
2.25	0.5406	3.019	5.74	0.6055
2.3	0.5344	3.085	6.005	0.5833
2.35	0.5286	3.149	6.276	0.5615
2.4	0.5231	3.212	6.553	0.5401
2.5	0.513	3.333	7.125	0.499
2.6	0.5039	3.449	7.72	0.4601
2.8	0.4882	3.664	8.98	0.3895
3	0.4752	3.857	10.33	0.3283
3.5	0.4512	4.261	14.12	0.2129
4	0.435	4.571	18.5	0.1388
4.5	0.4236	4.812	23.46	0.0917
5	0.4152	5	29	0.06172

Table 2: Normal shock, $\gamma = 1.4$.

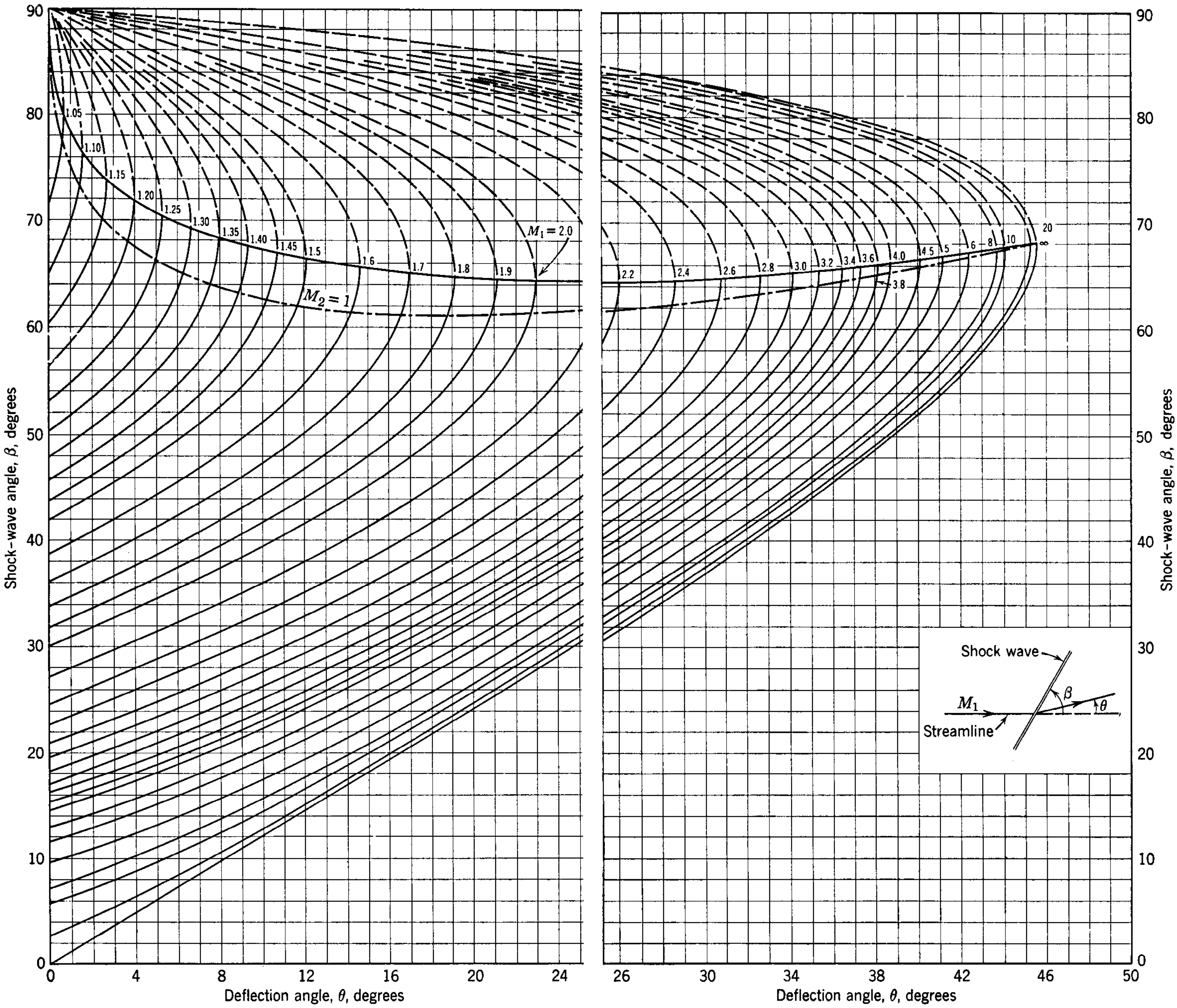

Figure 7: Oblique shock. Shock angle β vs. flow deflection angle θ for various upstream Mach numbers M_1. Perfect gas, $\gamma = 1.4$. (Liepmann and Roshko, 1957, pp. 428).

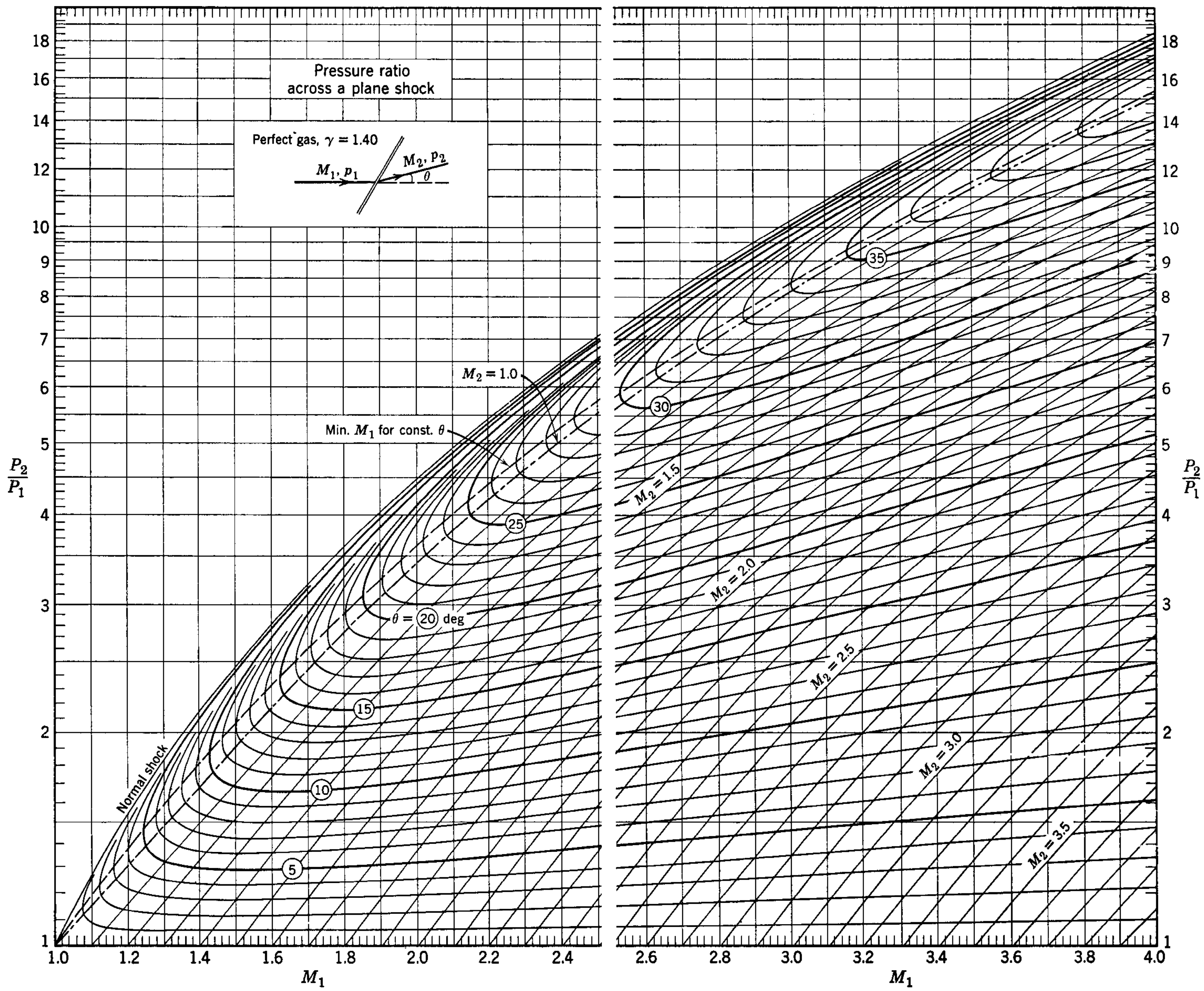

Figure 8: Oblique shock. Pressure ration p_2/p_1 and downstream Mach number M_2 vs. upstream Mach number for various flow deflection angles θ. (Liepmann and Roshko, 1957, pp. 430).