Pipe flow

In the laboratory unit on incompressible pipe flow the transition from laminar to turbulent flow is investigated. The pressure loss is measured under different conditions and the results are compared with known correlations from literature and with an analytical solution for laminar flow.

1 Basic principles

Dimensionless numbers The fully developed flow of a Newtonian fluid through a pipe of constant diameter can be unambiguously described by five quantities. These are the density ρ of the fluid, the mean velocity u, the dynamic viscosity μ, the inner diameter of the pipe d and the pressure gradient $\mathrm{d}p/\mathrm{d}x$. In a fully developed flow the properties of the flow do not change in flow direction, therefore a length L of the pipe does not play a role.

Two dimensionless numbers can be found from the five quantities given above. A common choice for these dimensionless numbers are the Reynolds number Re,

$$\mathrm{Re} = \frac{\rho u d}{\mu} = \frac{ud}{\nu}, \tag{1}$$

and the Darcy friction factor f,

$$f = \frac{(\mathrm{d}p/\mathrm{d}x)d}{\rho u^2/2}. \tag{2}$$

The Reynolds number is the ratio of inertial forces to viscous forces. The friction number relates the pressure loss along a length of the pipe of one diameter to the stagnation pressure. Moreover, from dimensional analysis follows that the friction number for hydraulically smooth pipes is a function *only* of the Reynolds number, since only two dimensionless numbers exist. For a rough pipe an additional dimensionless number must be added, the relative roughness ϵ/d, where ϵ is the mean depth of the roughness. The dependence between f and Re for various values of ϵ/d is often shown in the Moody-diagram, see the appendix.

Bernoulli's equation The conservation of mechanical energy (and of momentum) along a streamline yields Bernoulli's equation for inviscid, incompressible and steady flow,

$$p + \frac{\rho u^2}{2} + \rho g z = const. \tag{3}$$

To account for the loss of mechanical energy by friction, a pressure-loss term is added to Bernoulli's equation,

$$p_1 + \frac{\rho u_1^2}{2} + \rho g z_1 = p_2 + \frac{\rho u_2^2}{2} + \rho g z_2 + \Delta p_\mathrm{w}. \tag{4}$$

The pressure losses along a duct are added,

$$\Delta p_\mathrm{w} = \frac{\rho u^2}{2}\left(f\frac{L}{d} + \sum_i \zeta_i\right). \tag{5}$$

Here, the first term on the right-hand side describes the pressure loss due to friction, which follows from manipulation of eq. (2). The second term on the right-hand side gives pressure losses in built-in components such as fittings, valves or orifices, or also pressure losses in bends or manifolds. These pressure losses are reduced by the stagnation pressure and they are given in dimensionless form by the drag coefficient ζ.

Friction factor For laminar flow the pressure loss in the pipe depends linearly on the mean velocity u. For a tube with circular cross-section the law of Hagen-Poiseuille is valid, which may be written as

$$f = \frac{64}{\mathrm{Re}}. \tag{6}$$

In the double-logarithmic Moody-diagram eq. (6) yields a straight line with the inclination -1.

For turbulent flow through rough pipes f is nearly constant and independent of Re. In the transition region and for turbulent flow through hydraulically smooth pipes the value of f is given by correlations,

- Turbulent flow in smooth pipes (Prandtl equation)

$$\frac{1}{\sqrt{f}} = 2{,}0\log\left(\mathrm{Re}\sqrt{f}\right) - 0{,}8 \tag{7}$$

- Turbulent flow in rough pipes (Nikuradse)

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{3{,}71d}{k}\right) \tag{8}$$

- Transition region between smooth and rough pipes (Colebrook equation)

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{2{,}51}{\mathrm{Re}\sqrt{f}} + \frac{k}{3{,}71d}\right) \tag{9}$$

Since the above equations are *implicit* and hence not very practical, they can more easily be obtained from published charts. These charts are often referred to as Moody-diagram (see appendix). The Moody-diagram is a graph in non-dimensional form that relates the Darcy friction factor, Reynolds number and relative roughness for fully developed flow in a circular pipe. It can be used for working out pressure drop or flow rate down such a pipe.

Among many others an easy to use *explicit* approximation for turbulent flow in smooth pipes and $\mathrm{Re} < 10^5$ is the Blasius correlation

$$f = \frac{0{,}3164}{\mathrm{Re}^{0{,}25}}. \tag{10}$$

Transition from laminar to turbulent flow For Reynolds numbers $\lesssim 1800$ the flow through a pipe is always laminar. Above a critical value of Re the flow is instable against small disturbances. This critical value is, for the flow through a straight pipe with a circular cross-section, $\mathrm{Re} \approx 2300$. For higher Reynolds numbers the flow is usually turbulent. However, note that in experiments it has been possible to generate laminar flows with $\mathrm{Re} > 100.000$. The transition from laminar to turbulent flow is more or less abrupt and discontinuous.

Measurement devices Since the (volumetric or mass) flow rate is an extremely important parameter especially in industrial facilities, a great number of various measurement procedures and measurement devices exist. Among other measuring methods to determine the flow rate, mechanical flow meters, pressure-based meters (e.g., Venturi meter and Pitot tubes utilizing eq. (3)), optical flow meters, thermal mass flow meters, vortex flowmeters and electromagnetic, ultrasonic and Coriolis flow meters can be mentioned.

The same applies to pressure measurements, where many physical principles are being used and many instruments have been invented and designed. Pressure range, sensitivity, dynamic response and cost all vary by several orders of magnitude from one instrument design to the next.

In the lab we are going to use a differential pressure sensor (Validyne DP 103 Differential Pressure Transducer) which measures the displacement of a diaphragm (membrane) between two small chambers by means of changes in inductance.

A small selection of measurement principles and techniques will be presented and discussed in the laboratory course.

2 Test facility

In the laboratory course there is going to be a simple set up to run some experiments with water flowing through a straight pipe. As measuring devices the following tools are provided:

- an electronic balance (scale),
- a measuring tape,
- thermometer,
- stop watch (or use mobile phone),
- a differential pressure sensor (Validyne DP103)
- a PC incl. Easy-Sense software for data acquisition and processing
- the tables and the Moody-diagram provided in the appendix.

Important notes:

- The pressure sensor is calibrated to Pascal! $1\,\mathrm{Pa} = 1\,\mathrm{N/m^2} = 1\,\mathrm{kg/(m \cdot s^2)}$.
- To measure the velocity of the flow, use $\dot{m} = m/t = \rho u A$, where m denotes the mass [kg] and t the time [s], respectively.

3 Exercises and questions

Please use the same successive numbering (a, b, c . . .) for your report as below!

a.) Sketch the experimental setup including lengths. Explain the principle with a few words.

b.) Which height H of the water level (in the vessel) above the pipe is needed to get a Reynolds number of 1600 in the pipe? (Use eq. (3) to calculate a rough estimate by neglecting pressure losses).

c.) Set the calculated height H and then check the Reynolds number with the available tools (see section 2).

d.) Try to set the Reynolds number in the pipe to 1600 as exactly as possible. Which height is actually needed? Which pressure loss in the pipe (per meter) do you estimate by this?

e.) Measure the pressure loss in the pipe over the measuring distance L for 3 different

Reynolds numbers (e.g., 1200, 1400, 1600).

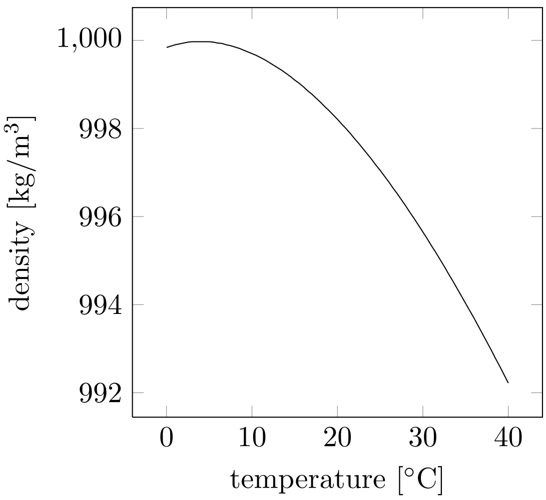
f.) Compare the values from e.) with the values of f in the Moody-diagram.

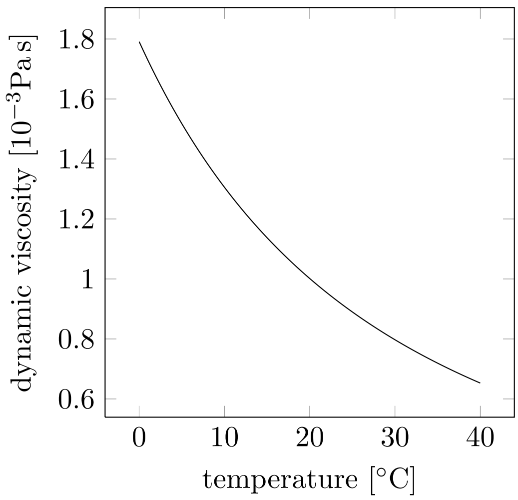
g.) Trigger and visualize turbulent spots with ink. Describe/sketch in short what you see. How does turbulence influence the outflow angle?

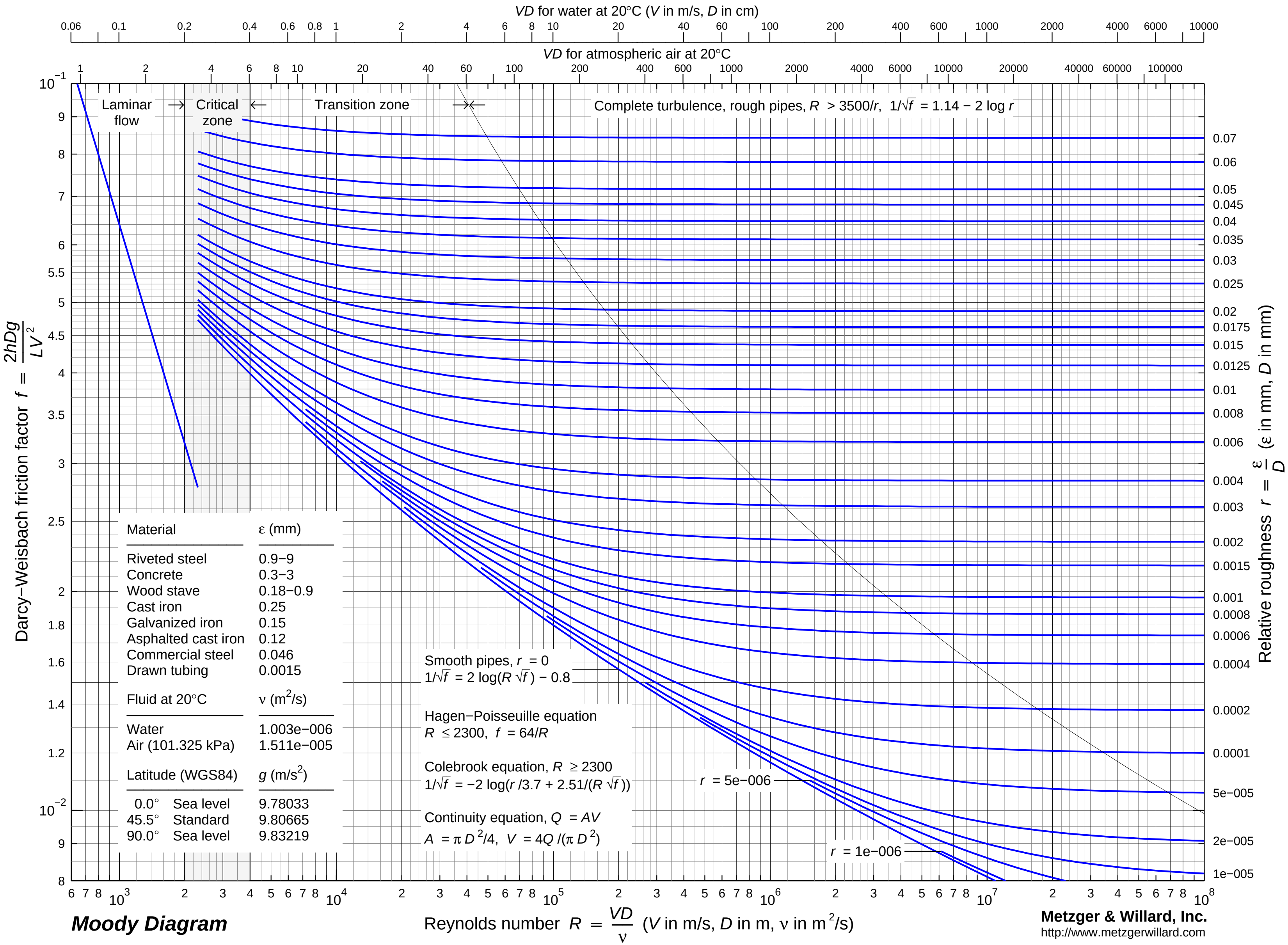
h.) Set the Reynolds number to the turbulent regime (e.g. 4000). Measure the pressure loss and compare the value to eq. (10) and the moody diagram.

4 Appendix

Density and dynamic viscosity of water in dependence of temperature. Data from http://webbook.nist.gov.







VD for water at 20°C (V in m/s, D in cm)
VD for atmospheric air at 20°C
Laminar flow
Critical zone
Transition zone
Complete turbulence, rough pipes, R > 3500/r, 1/√f = 1.14 − 2 log r
Darcy−Weisbach friction factor f = 2hDg/LV²
Relative roughness r = ε/D (ε in mm, D in mm)
Material | ε (mm)
Riveted steel | 0.9−9
Concrete | 0.3−3
Wood stave | 0.18−0.9
Cast iron | 0.25
Galvanized iron | 0.15
Asphalted cast iron | 0.12
Commercial steel | 0.046
Drawn tubing | 0.0015
Fluid at 20°C | ν (m²/s)
Water | 1.003e−006
Air (101.325 kPa) | 1.511e−005
Latitude (WGS84) | g (m/s²)
0.0° Sea level | 9.78033
45.5° Standard | 9.80665
90.0° Sea level | 9.83219
Smooth pipes, r = 0
1/√f = 2 log(R √f) − 0.8
Hagen−Poiseuille equation
R ≤ 2300, f = 64/R
Colebrook equation, R ≥ 2300
1/√f = −2 log(r/3.7 + 2.51/(R √f))
Continuity equation, Q = AV
A = π D²/4, V = 4Q/(π D²)
r = 5e−006
r = 1e−006
Moody Diagram
Reynolds number R = VD/ν (V in m/s, D in m, ν in m²/s)
Metzger & Willard, Inc.
http://www.metzgerwillard.com